

Calhoun Community College

1st Annual Mathematics Tournament

December 4, 2015



Individual Test

Please read each problem carefully. After working the problem, select the correct answer from the five choices and mark that answer on your Scantron. In the event of a tie (for individual awards), the following starred (*) test items will serve as tie breakers in the order listed: 5, 10, 13, 15, 19, 20, 27, 30, and 31. There is no correction factor for guessing and any question left blank will be scored as an incorrect. You will have 50 minutes to finish the test. The test is machine graded so make sure to use a #2 pencil that will be provided and if you need to change an answer, make sure to erase completely.

1. Two fair dice are rolled. What is the probability of rolling a sum greater than 10?

- a) $\frac{1}{12}$ b) $\frac{1}{18}$ c) $\frac{1}{36}$ d) $\frac{1}{9}$ e) $\frac{7}{36}$

2. If $\frac{a}{b+c+d} = \frac{4}{3}$ and $\frac{a}{b+c} = \frac{3}{5}$, then the value of $\frac{d}{a}$ is

- a) $\frac{7}{6}$ b) $\frac{6}{7}$ c) $\frac{-12}{11}$ d) $\frac{-11}{12}$ e) $\frac{15}{11}$

3. Let $f(x)$ be a quadratic polynomial such that $f(3) = 15$ and $f(-3) = 9$. Find the coefficient of x in $f(x)$.

- a) 2 b) 3 c) -3 d) 1 e) none of these

4. How many integers are there such that $7x + 2 \leq 23$ and $3x - 5 \geq 1$?

- a) 0 b) 1 c) 2 d) 3 e) more than 3

5. *Find y' if $y = \sin(\tan\sqrt{\sin x})$.

- a) 0 b) 1 c) $\frac{\cos(\tan\sqrt{\sin x}) \sec^2 x \cos x}{2}$ d) $\frac{\frac{5}{2} \sin^{\frac{3}{2}} \cos x + \sin^{\frac{7}{2}} x}{\cos^2 x}$
e) $\frac{\cos(\tan\sqrt{\sin x}) \sec^2(\sqrt{\sin x}) \cos x}{2\sqrt{\sin x}}$

6. Find the value of x that satisfies the equation $2^{\log_4(8)} = 16^x$.

- a) $\frac{3}{8}$ b) $\frac{1}{2}$ c) $2^{3/32}$ d) $2^{1/8}$ e) none of these

7. Solve:

$$|m + 2| + 10 \leq 25$$

- a.) $\{m \mid -17 \leq m \leq 13\}$
b.) $\{m \mid -37 < m < 13\}$
c.) $\{m \mid -17 < m < 13\}$
d.) $\{m \mid -37 \leq m \leq 13\}$
e.) The solution set is all real numbers

8. $ABCD$ is a parallelogram. $AB = BC = 20$ and $m\angle ABC = 120^\circ$. Calculate the area of the parallelogram $ABCD$.

- a) $100\sqrt{2}$ b) $100\sqrt{3}$ c) $400\sqrt{2}$ d) $400\sqrt{3}$ e) none of these

9. If the dot product of two vectors is $-5\sqrt{3}$ and the angle between them is $\frac{5\pi}{6}$, then what is the product of the magnitudes of the two vectors?

- a) -2 b) 10 c) 2 d) 5 e) -10

10. *If $\sin x + \cos x = \frac{1}{3}$, what is the value of $\sin^3 x + \cos^3 x$?

- a) $\frac{1}{9}$ b) $\frac{4}{3}$ c) $\frac{13}{27}$ d) $\frac{5}{9}$ e) $\frac{8}{27}$

11. Suppose $f(x) = ax + b$ and a and b are real numbers. We define

$$f_1(x) = f(x) \text{ and } f_{n+1}(x) = f(f_n(x))$$

for all positive integers n . If $f_7(x) = 128x + 381$, what is the value of $b - a$?

- a) 1 b) 2 c) 3 d) 4 e) 5

12. Given that $f(x) = 2x^3 - 3x + 4$, find t such that $f^{-1}(t) = 0.4$

- a) 1 b) $\sqrt[3]{3.064}$ c) -4 d) 5.328 e) 2.928

13. *Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$

- a) 0 b) 1 c) ∞ d) -1 e) none of these

14. Find the exact value of the expression $\cos \left[2 \tan^{-1} \left(-\frac{4}{3} \right) \right]$.

- a) $\frac{3}{5}$ b) $-\frac{7}{25}$ c) $-\frac{6}{5}$ d) $\frac{3}{8}$ e) none of these

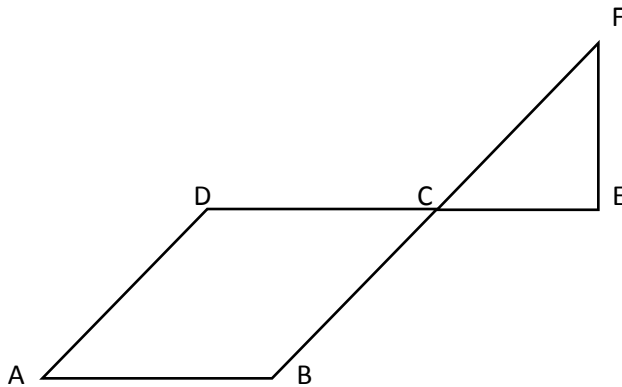
15. *An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those six years is approximated by

$$\frac{dh}{dt} = 3.5t + 9,$$

where t is the time in years and h is the height in centimeters. The seedlings are 15 centimeters tall when planted ($t=0$). How tall are the shrubs when they are sold?

- a) 165 cm b) 78 cm c) 132 cm d) 270 cm e) 117 cm

16. Given that $ABCD$ is a parallelogram with $m\angle A = 45^\circ$, find the area of triangle CEF if $AB = 30$, C is the midpoint of BF , and the area of $ABCD$ is 600 units².



- a) 100 units² b) 200 units² c) 300 units² d) 250 units² e) none of these

17. Find the partial fraction decomposition:

$$\frac{x - 18}{x(x - 3)^2}$$

- a) $-\frac{2}{x} - \frac{11}{(x-3)^2}$ b) $\frac{2}{x} - \frac{2}{(x-3)} - \frac{5}{(x-3)^2}$ c) $-\frac{2}{x} + \frac{2}{(x-3)} - \frac{5}{(x-3)^2}$
 d) $\frac{2}{x} - \frac{2x-11}{(x-3)^2}$ e) $-\frac{2}{x} + \frac{5}{(x-3)^2}$

18. If $3\sin\theta + 4\cos\theta = 5$, then $\tan\theta$ is

- a) 1 b) -1 c) $\frac{3}{4}$ d) $\frac{4}{3}$ e) 0

19. *The sum of the perimeters of an equilateral triangle and a square is 19. Find the dimensions of the triangle and the square that produce a minimum total area.

a. Side of square: $\frac{19}{4+9\sqrt{3}}$; Side of triangle $\frac{57\sqrt{3}}{4+9\sqrt{3}}$

b. Side of square: $\frac{19\sqrt{3}}{9+4\sqrt{3}}$; Side of triangle $\frac{57}{9+4\sqrt{3}}$

c. Side of square: $\frac{57\sqrt{3}}{4+9\sqrt{3}}$; Side of triangle $\frac{19}{4+9\sqrt{3}}$

d. Side of square: $\frac{19}{9+4\sqrt{3}}$; Side of triangle $\frac{38\sqrt{3}}{9+4\sqrt{3}}$

e. Side of square: $\frac{57}{9+4\sqrt{3}}$; Side of triangle $\frac{19}{9+4\sqrt{3}}$

20. *A music venue is trying to determine what price to charge for tickets. At a price of \$12 per ticket, it averages 300 people for a performance of Microwave Fave and the Nukes. Every decrease of \$1 in the ticket price adds 25 people to the average number attending. Every person at the concert spends an average of \$8 on concessions. How many people will attend at the price that will maximize the revenue?

- a) 450 b) 500 c) 550 d) 600 e) none of these

21. What is the value of $(\log_{11} 12)(\log_{12} 13) \dots (\log_{99} 100)$?

- a) e b) 10 c) 2 d) $\frac{1}{2}$ e) none of these

22. What is the exact value of

$$4 \sin^2 50^\circ \cos^2 50^\circ + \sin^4 50^\circ + \cos^4 50^\circ - 2 \cos^2 50^\circ \sin^2 50^\circ$$

- a) 0 b) 1 c) $\frac{\sqrt{2-\sqrt{3}}}{2}$ d) $\frac{\sqrt{2+\sqrt{3}}}{2}$ e) $\frac{\sqrt{3+\sqrt{2}}}{3}$

23. In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge based on the following facts:

- The Ford finished 7 seconds before the Chevrolet.
- The Pontiac finished 6 seconds after the Buick.
- The Dodge finished 8 seconds after the Buick.
- The Chevrolet finished 2 seconds before the Pontiac.

List the finishers from fifth to first.

- Ford, Pontiac, Chevrolet, Buick, Dodge
- Pontiac, Chevrolet, Buick, Dodge, Ford
- Dodge, Pontiac, Chevrolet, Buick, Ford
- Ford, Buick, Dodge, Pontiac, Chevrolet
- None of These

24. When a polynomial $P(x)$ is divided by $x - 2$, the quotient is $3x^2 - x - 5$ and the remainder is -1 . Then $P(-1)$ equals

- 2
- 3
- 4
- 7
- None of these

25. Find $y = f(x)$, if $f''(x) = x^2$, $f'(0) = 7$ and $f(0) = 2$.

- $y = -\frac{1}{12}x^4 + 7x + 2$
- $y = \frac{1}{12}x^4 + 7x + 2$
- $y = \frac{1}{12}x^4$
- 0
- 1

26. Find the sum of all values of x so that $16^{(x^2+3x-1)} = 8^{(x^2+3x+2)}$.

- 1
- 5
- 3
- 3
- none of these

27. *If a , b , and c are roots of the equation $x^3 - 3x + 7 = 0$, compute the numerical value of

$$(a + 1)(b + 1)(c + 1).$$

- 9
- 4
- 4
- 5
- 11

28. A frog is in a well 12 feet deep. Each day the frog climbs up 3 feet, and each night it slips back 2 feet. How many days will it take the frog to get out of the well?

- a) 12 days b) 10 days c) 14 days d) 11 days e) none of these

29. Suppose that 1 and -7 are roots of $x^3 + ax^2 + bx + c = 0$, and that $a + b = -15$. Find the final root.

- a) 4 b) 2 c) 8 d) 0 e) none of these

30. *Evaluate $\int \tan \theta \sec \theta d\theta$.

- a) $\frac{1}{\cos \theta} + C$ b) $\frac{\tan^2 \theta}{2} + C$ c) $\frac{\sec^2 \theta}{2} + C$ d) $\frac{1}{\sin \theta} + C$ e) $\frac{\tan^2 \theta \sec^2 \theta}{2} + C$

31. *A conical tank (with vertex down) is 12 feet across the top and 18 feet deep. If water is flowing into the tank at a rate of 18 cubic feet per minute, find the rate of change of the depth of the water when the water is 10 feet deep.

- a) $\frac{9}{40\pi}$ ft/min b) $\frac{9}{100\pi}$ ft/min c) $\frac{81}{20\pi}$ ft/min d) $\frac{81}{50\pi}$ ft/min e) $\frac{81}{200\pi}$ ft/min

32. Find the inverse of the function:

$$f(x) = \frac{6x+7}{x-1}$$

- a) $f^{-1}(x) = \frac{x-1}{6x+7}$ b) $f^{-1}(x) = \frac{x+7}{x-6}$ c) $f^{-1}(x) = \frac{x+1}{6x-7}$ d) $f^{-1}(x) = \frac{6x-7}{x+1}$ e) $f^{-1}(x) = \frac{x-7}{x+6}$

33. Let \otimes be an operation defined on functions such that:

$$f \otimes g(x) = (f(g(x))) (g(f(x)))$$

If $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find $f \otimes g(x)$.

- a) $6x^2 - 12x + 10$ b) $8x^4 - 28x^3 + 20x^2 + 12x - 10$ c) $8x^4 - 24x^3 + 16x^2 + 12x - 10$
d) $8x^4 - 4x^3 - 4x^2 + 2x$ e) $8x^4 + 16x^2 - 10$