

# **2<sup>nd</sup> ANNUAL**

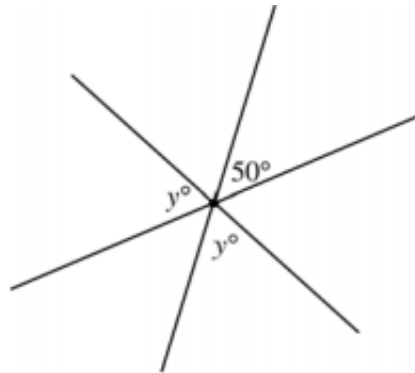


**FRIDAY, December 2**

## **Individual Exam**

Please read each problem carefully. After working the problem, select the correct answer from the five choices and mark that answer on your scantron. In the event of a tie (for individual awards), the following starred (\*) test items will serve as tie breakers in the order listed: 5, 10, 13, 15, 19, 20, 21, 27, and 32. There is no correction factor for guessing and any question left blank will be scored as incorrect. You will have 50 minutes to complete the exam. The scantron sheet is machine graded so be sure to use the provided #2 pencil and if you need to change an answer, erase your former selection completely.

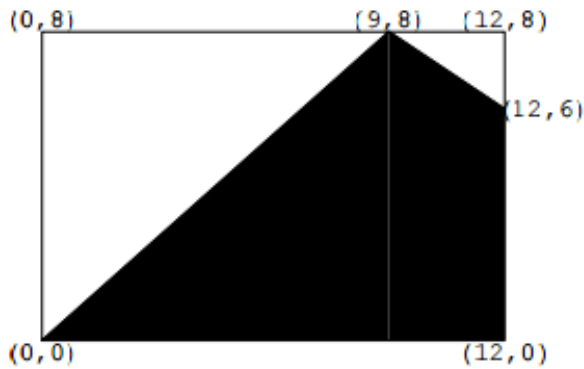
1. In the following figure,



three lines intersect at a point. What is the value of  $y$ ?

- a) 80      b) 75      c) 70      d) 65      e) none of these

2. In the following figure, vertices are labeled as points. Find the area of the shaded region.



- a) 18      b) 24      c) 56      d) 96      e) none of these

3. The following data set represents the number of vacation days used by a sample of 20 employees in a recent year. Find the data set's first, second, and third quartiles.

3    9    2    1    7    5    3    2    2    6  
 4    0    10    0    3    5    7    8    6    5

- a) 2.5, 4.0, 6.0  
 b) 2.25, 4.25, 6.75  
 c) 2.0, 4.5, 6.5  
 d) 2.75, 4.75, 6.25  
 e) none of these

4. Solve:  $e^{2x} - 2e^x - 35 = 0$
- a)  $\ln 7, \ln 5$       b)  $\ln 7$       c)  $\ln 5$       d)  $\ln 7, \ln (-5)$       e)  $\ln (-7), \ln 5$
5. \*A cylindrical gasoline tank is placed so that the axis of the cylinder is horizontal. Find the fluid force on a circular end of the tank if the tank is half full, assuming that the diameter is 9 feet and the gasoline weighs 42 pounds per cubic foot.
- a) 1276 lbs.      b) 20,412 lbs.      c) 5,103 lbs.      d) 2,552 lbs.      e) 10,206 lbs.
6. How many distinguishable permutations of letters are possible using the letters in the word COMMITTEE?
- a) 45,360      b) 90,720      c) 181,440      d) 362,880      e) none of these
7. A survey of 136 college students was done to find out what elective courses they were taking. Let  $A$  = the set of those taking art;  $B$  = the set of those taking basket weaving; and  $C$  = the set of those taking canoeing. The study revealed the following information:  
 $n(A) = 45$ ;  $n(B) = 55$ ;  $n(C) = 40$ ;  
 $n(A \cap B) = 12$ ;  $n(A \cap C) = 15$ ;  $n(B \cap C) = 23$ ;  
 $n(A \cap B \cap C) = 2$ .  
How many students were not taking any of these electives?
- a) 54      b) 46      c) 44      d) 10      e) none of these
8. What is the sum of the zeros of the function  $f(x) = 9x^4 - 9x^3 + 83x^2 - 81x + 18$ ?
- a)  $-1$       b) 1      c) 7      d)  $-7$       e) none of these
9. Let  $f(x) = (x - b)/(x - a)$  for constants  $a$  and  $b$ . If  $f(2) = 0$  and  $f(1)$  is undefined, what is  $f(1/2)$ ?
- a) 0      b) 2      c)  $1/3$       d) 3      e) none of these

10. \*Find the point of the graph of  $f(x) = x^2$  that is closest to the point  $(2, \frac{1}{2})$ .

- a) (2,4)      b) (1,1)      c)  $(\frac{1}{2}, \frac{1}{4})$       d) (0,0)      e)  $(1, \frac{1}{2})$

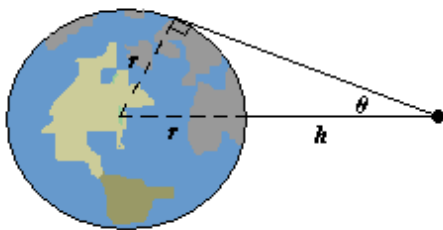
11. A bag contains 3 blue, 4 red, and 4 green marbles. Four marbles are drawn at random from the bag. How many different samples are possible which include exactly two red marbles?

- a) 6      b) 504      c) 126      d) 18      e) none of these

12.  $U = \{1,2,3,4,5,6,7,8,9,10\}$ ,  $A = \{1,3,5,7\}$  and  $B = \{2,4,6,8\}$ . Find  $(A \cup B)'$ .

- a)  $\{1,2,3,4,5,6,7,8\}$       b)  $\{1,2,3,4,5,6,7,8,9,10\}$       c)  $\{9,10\}$       d)  $\{\}$       e) none of these

13. \*When satellites observe Earth, they can scan only part of Earth's surface. Some satellites have sensors that can measure the angle  $\theta$  shown in the figure. Let  $h$  represent the satellite's distance from Earth's surface and let  $r$  represent Earth's radius. Find the rate at which  $h$  is changing with respect to  $\theta$  when  $\theta = 30^\circ$ . (Assume  $r = 4050$  miles.) Round your answer to the nearest unit.



- a) 14030 mi/radian  
b) -14030 mi/radian  
c) -8100 mi/radian  
d) 8100 mi/radian  
e) none of these

14. Consider the polynomial  $f(x) = x^4 - 14x^3 + 71x^2 - 154x + 120$ . What is the minimum number of imaginary roots of  $f(x)$ ?

- a) at least 1 imaginary root  
b) at least 2 imaginary roots  
c) at least 3 imaginary roots  
d) at least 4 imaginary roots  
e) no imaginary roots

15. \*Find the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis.

$$y = \sin x, y = 0, x = 0, x = \frac{\pi}{3}$$

a)  $\frac{1}{3}\pi^2 - \frac{\sqrt{3}}{8}\pi$

b)  $\frac{1}{6}\pi^2 + \frac{\sqrt{3}}{8}\pi$

c)  $\frac{1}{6}\pi^2 - \frac{\sqrt{3}}{8}\pi$

d)  $\frac{1}{3}\pi^2 + \frac{\sqrt{3}}{8}\pi$

e)  $\frac{1}{6}\pi^2 - \frac{\sqrt{3}}{4}\pi$

16. The odds against a spinner landing on blue are 4:3. Find the probability that the spinner lands on blue.

a)  $\frac{4}{3}$

b)  $\frac{4}{7}$

c)  $\frac{3}{7}$

d)  $\frac{3}{4}$

e) 1

17. Determine the open interval on which the graph of  $y = -x^3 + 6x^2 - 2$  is concave upward.

a)  $(-\infty, 2)$

b)  $(-\infty, -2)$

c)  $(2, \infty)$

d)  $(-2, \infty)$

e) none of these

18. The ages of the winners of a cycling tournament are approximately bell-shaped. The mean age is 27.3 years, with a standard deviation of 3.6 years. The winner in one recent year was 29 years old. Transform the age to a z-score and determine if the age is unusual based on the given criteria in the problem.

a) No, this age is not unusual. A z-score between -2 and 2 is not unusual.

b) Yes, this age is unusual. A z-score between -2 and 2 is unusual.

c) No, this age is not unusual. A z-score outside of the range from -2 to 2 is not unusual.

d) Yes, this age is unusual. A z-score outside of the range from -2 to 2 is unusual.

e) none of these

19. \*A local television station sends out questionnaires to determine if viewers would rather see a documentary, an interview show, or reruns of a game show. There were 250 responses with the following results:

75 were interested in an interview show and a documentary, but not reruns;

10 were interested in an interview show and reruns, but not a documentary;

35 were interested in reruns but not an interview show;

60 were interested in an interview show but not a documentary;

25 were interested in a documentary and reruns;

15 were interested in an interview show and reruns;

20 were interested in none of the three.

How many are interested in exactly one kind of show?

- a) 100                  b) 110                  c) 120                  d) 130                  e) none of these

20. \*Evaluate the following improper integral:  $\int_{10}^{\infty} \int_0^{1/x} y \, dy \, dx$

- a)  $\frac{121}{20}$                   b)  $\frac{11}{20}$                   c)  $\frac{20}{11}$                   d)  $\frac{20}{121}$                   e) does not converge

21. \*Find the four positive integers,  $a$ ,  $b$ ,  $c$ , and  $d$ , such that  $a + b + c + d$  is the smallest sum possible satisfying

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 5 & -10 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

- a) 4                  b) 8                  c) 12                  d) 16                  e) 24

22. The square  $ABCD$  has sides of length 10. Points  $J$ ,  $K$ ,  $L$ , and  $M$  are placed on sides  $AB$ ,  $BC$ ,  $CD$ , and  $DA$  so that  $AJ = BK = CL = DM = 2$ . What is the area of the quadrilateral  $JKLM$ ?

- a) 48                  b) 52                  c) 56                  d) 64                  e) 68

23. Let  $f(x) = \sin(\pi x)$  and let  $g(x) = [f(x)]^2$ . Find  $f\left(f\left(\frac{1}{6}\right)\right) + g\left(g\left(\frac{1}{3}\right)\right)$ .

- a)  $\frac{3}{2}$       b) 1      c)  $\frac{4}{3}$       d)  $\frac{3}{4}$       e) 2

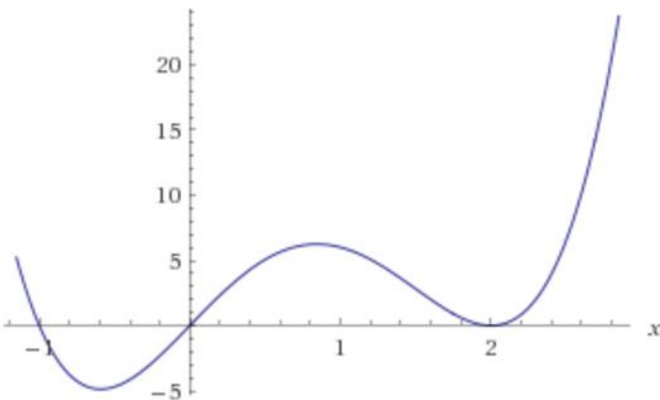
24. Simplify  $\cos x + \cos(\pi - x)$ .

- a)  $2 \cos x$       b)  $\cos x - \sin x$       c)  $\cos x + \sin x$       d) 0      e) none of these

25. The pressure of a gas varies jointly as the amount of the gas (measured in moles) and the temperature and inversely as the volume of the gas. If the pressure is 1350 kPa (kiloPascals) when the number of moles is 7, the temperature is 300° Kelvin, and volume is 560cc, find the pressure when the number of moles is 10, the temperature is 290°K, and the volume is 600cc.

- a) 1740      b) 870      c) 1560      d) 960      e) none of the above

26. Find the equation of the 4<sup>th</sup> degree polynomial with the following graph:  
(Hint  $f(3) = 36$ ).



- a)  $f(x) = \frac{6}{25}x^4 + \frac{18}{25}x^3 - \frac{24}{25}x$   
 b)  $f(x) = x^4 - 3x^3 + 4x$   
 c)  $f(x) = x^4 - 6x^3 + 13x^2 - 12x + 4$   
 d)  $f(x) = 3x^4 - 9x^3 + 12x$   
 e) none of these

27. \*Find the sum of the convergent series  $\sum_{n=1}^{\infty} (-1)^n \frac{2}{(n+8)(n+10)}$

- a)  $\frac{1}{72}$       b)  $-\frac{3}{8}$       c)  $\frac{2}{7}$       d)  $\frac{1}{90}$       e) none of these

28. A ball is thrown vertically upwards from a height of 6 ft with an initial velocity of 60 ft per second. How high will the ball go?

- a) 174.75 ft      b) 178.75 ft      c) 62.25 ft      d) 162.75 ft      e) none of these

29. Which quadrilateral does not always have congruent diagonals?

- a) square      b) rectangle      c) isosceles trapezoid      d) rhombus      e) none of these

30. Simplify  $i^{12345}$ .

- a) 1      b) -1      c)  $i$       d)  $-i$       e) none of these

31. Evaluate the determinant:  $\begin{vmatrix} 5 & 2 & 5 \\ 4 & 2 & 3 \\ 2 & 6 & 4 \end{vmatrix}$

- a) -6      b) 314      c) -30      d) 6      e) none of these

32. \*Find the interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{x^{3n+1}}{(3n+1)!}$ .

- a)  $(-\infty, 4)$       b)  $[-4, 4]$       c)  $[-3, 3]$       d)  $(4, \infty)$       e)  $(-\infty, \infty)$

33. Find the limit:  $\lim_{x \rightarrow 0^-} \left( x^4 - \frac{1}{x} \right)$

- a) 0      b)  $\infty$       c) 1      d) -1      e)  $-\infty$