

3rd ANNUAL



FRIDAY, December 8

Individual Exam

Please read each problem carefully. After working the problem, select the correct answer from the five choices and mark that answer on your scantron. In the event of a tie (for individual awards), the following starred (*) test items will serve as tie breakers in the order listed: 5, 7, 9, 12, 13, 21, 26, 29, and 30. There is no correction factor for guessing and any question left blank will be scored as incorrect. You will have 50 minutes to complete the exam. The scantron sheet is machine graded so be sure to use the provided #2 pencil and if you need to change an answer, erase your former selection completely.

1. Find all real solutions to the following logarithmic equation:

$$\log(x^2 - 20) = \log\left(\frac{1}{2}\right) + \log(-2x)$$

- A) $\{-4,5\}$
B) $\{-2,10\}$
C) $\{4\}$
D) $\{5\}$
E) None of these
2. Find the vertices of the ellipse given by $25x^2 + 9y^2 + 200x - 90y + 400 = 0$.
- A) $(4,0), (4,-10)$
B) $(4,8), (4,2)$
C) $(-1,5), (-1,5)$
D) $(1,5), (1,5)$
E) $(-4,10), (-4,0)$
3. How much work is done pulling a wagon up an incline of 30 degrees for 60 feet, then a level stretch of 250 feet, and then another incline of 60 degrees for 50 feet if a constant force of 30 pounds is used?
- A) 5688 ft-lbs
B) 6980 ft-lbs
C) 9699 ft-lbs
D) 10,855 ft-lbs
E) none of these
4. Find the maximum value of the function and the values of x and y at which the maximum occurs.
 $G = 7x - 2y$, subject to: $x \geq 0$
 $0 \leq y \leq 7$
 $x - y \leq 4$
 $x + 2y \leq 16$
- A) Maximum -14 occurring at $(0, 7)$
B) Maximum 48 occurring at $(8, 4)$
C) Maximum 0 occurring at $(2, 7)$
D) Maximum 28 occurring at $(4, 0)$
E) none of these

5. *A cylindrical water tank 5 meters high with a radius of 2 meters is buried so that the top of the tank is 1 meter below ground level. How much work (in joules) is done in pumping a full tank of water up to ground level? (Water weighs 9800 newtons per cubic meter.)

- A) $548,800\pi$ J B) $686,000\pi$ J C) $411,600\pi$ J D) $1,234,800\pi$ J E) $1,209,000\pi$ J

6. Let $f(x) = x^4 - 4x^3 + 1$. Find the open interval(s) where the function is concave up.

- A) $(-\infty, 0)$ B) $(2, \infty)$ C) $(-\infty, 0) \cup (2, \infty)$ D) $(0, 2)$ E) none of these

7. *Find the average value of $f(x, y) = e^{x+y}$ over the region R , where R is a triangle with vertices $(0,0)$, $(0,9)$ and $(9,9)$.

- A) $\frac{1}{81}(e^9 - 1)^2$ B) $\frac{1}{81}(e^9 + 1)^2$ C) $(e^9 - 1)^2$ D) $(e^9 + 1)^2$ E) $\frac{1}{2}(e^9 - 1)^2$

8. Evaluate $\left(\cos \frac{\pi}{12} - \cos \frac{5\pi}{12} \right) \left(\sin \frac{\pi}{12} - \sin \frac{5\pi}{12} \right)$.

- A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$ E) none of these

9. *Find the Maclaurin polynomial of degree 4 for the function $f(x) = e^{9x}$.

- A) $1 + 9x + \frac{81}{2}x^2 + \frac{729}{4}x^3 + \frac{2187}{2}x^4$
 B) $1 + 9x + \frac{27}{2}x^2 + \frac{243}{4}x^3 + \frac{2187}{16}x^4$
 C) $1 - 9x + \frac{81}{2}x^2 - \frac{243}{2}x^3 + \frac{2187}{8}x^4$
 D) $1 - 9x + \frac{27}{2}x^2 - \frac{243}{4}x^3 + \frac{2187}{16}x^4$
 E) $1 + 9x + \frac{81}{2}x^2 + \frac{243}{2}x^3 + \frac{2187}{8}x^4$

10. The distance between points $(-3,2)$ and $(c,-6)$ is $4\sqrt{5}$. Find all the possible values of c .

- A) $c = 3$ B) $c = -7, c = 1$ C) $c = 3, c = -9$ D) $c = 1$ E) none of these

11. Given that $f(x) - 2f(10 - x) = x^2$ for all real number values of x , compute the value of $f(2)$.

- A) 4 B) $-\frac{56}{3}$ C) -24 D) $-\frac{124}{3}$ E) -44

12. *Find the limit: $\lim_{x \rightarrow 3} \frac{\sqrt{x+6}-3}{x^3-3x^2}$

- A) 0 B) $-\frac{5}{54}$ C) $\frac{1}{54}$ D) $\frac{1}{3}$ E) none of these

13. *Because of the high cost of living, Kimberly, Terry, and Otis each hold down two jobs, but no two have the same occupation. The occupations are doctor, engineer, teacher, lawyer, writer, and painter. Given the following information, determine the occupations of each individual:

- a.) The doctor had lunch with the teacher.
- b.) The teacher went fishing with Kimberly, who is not the writer.
- c.) The painter is related to the engineer.
- d.) The doctor hired the painter to do a job.
- e.) Terry lives next door to the writer.
- f.) Otis beat Terry and the painter at tennis.
- g.) Otis is not the doctor.

- A) Terry is the lawyer and the painter; Kimberly is the engineer and the doctor; Otis is the teacher and the writer.
- B) Kimberly is the teacher and the engineer; Terry is the lawyer and the doctor; Otis is the writer and the painter.
- C) Kimberly is the lawyer and the painter; Terry is the engineer and the doctor; Otis is the teacher and the writer.
- D) Otis is the engineer and the lawyer; Kimberly is the doctor and teacher; Terry is the painter and the writer.
- E) Otis the lawyer and the painter; Terry is the teacher and the writer; Kimberly is the engineer and the doctor.

14. How many two-digit prime numbers are there in which both digits are prime numbers?

- A) 15 B) 8 C) 5 D) 4 E) 3

15. A single fair die is rolled. Find the odds in favor of getting some number greater than 5.
- A) 1:6 B) 1:5 C) 5:1 D) 3:4 E) 6:1
16. Find the equation of the line that is tangent to the graph of $f(x) = 3x^2 - 2$ and parallel to the line $6x - y = 7$
- A) $6x + y = 7$
B) $6x - y = 5$
C) $6x - y = 2$
D) $x - 6y = -5$
E) none of these
17. Tom found the value of $3^{21} = 10,4A0,353,20B$. He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?
- A) 0 B) 2 C) 3 D) 6 E) none of these
18. Solve for z given $2z + 5 = 6 - iz$.
- A) $0.4 - i$ B) $0.8 + 0.6i$ C) $0.8 - 0.6i$ D) $0.6 - 0.8i$ E) $0.4 - 0.2i$
19. An outdoor Christmas tree vendor sells an average of 50 trees per day in dry weather and 15 per day in wet weather. If the weather in this area is wet 25% of the time, determine the expected (average) number of Christmas trees sold per day.
- A) 23.75 B) 38.5 C) 32.5 D) 41.25 E) none of these
20. Every day, the value of a stock rises by exactly two dollars in the morning, and then falls by exactly one dollar in the afternoon. If the stock's value at the start of the first day is \$100, on what day will the stock's value first reach \$200?
- A) 50 B) 99 C) 100 D) 101 E) 200

21. *Find the limit: $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{\sin(\frac{1}{n})}{n}}$

- A) does not exist B) 1 C) 0 D) ∞ E) none of these

22. How many pairs (x,y) of integers satisfy $x^4 - y^4 = 16$?

- A) infinitely many B) 4 C) 2 D) 1 E) 0

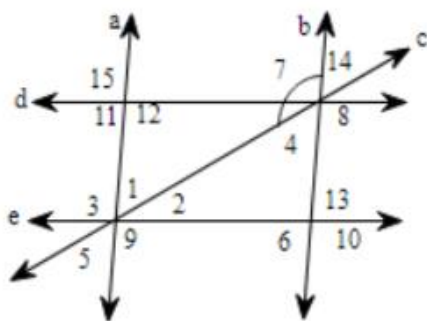
23. Two of three sides of a triangle are 3.2 cm and 5.5 cm. Which of the following is not a possible perimeter of the triangle?

- A) 10.9 B) 12.2 C) 17.3 D) 15.6 E) 14.7

24. In the Clock Game, part of the game show *The Price Is Right*, a contestant must guess the price (rounded to the nearest dollar) of a prize which is worth less than \$2,000. After each guess, the contestant is told whether her guess was correct, too low, or too high. Assume that the contestant is mathematically savvy, but has no idea how much the prize is worth. With how many guesses is she guaranteed to win the prize?

- A) 10 B) 11 C) 12 D) 13 E) 199

25. Use the following figure to identify all pairs of corresponding angles for the lines d and e with transversal a .



- A) $\angle 1$ & $\angle 5$, $\angle 15$ & $\angle 3$
 B) $\angle 3$ & $\angle 11$, $\angle 15$ & $\angle 1$
 C) $\angle 15$ & $\angle 3$, $\angle 12$ & $\angle 1$
 D) $\angle 12$ & $\angle 9$, $\angle 15$ & $\angle 11$
 E) $\angle 12$ & $\angle 9$, $\angle 15$ & $\angle 3$

26. *The trigonometric expression $\frac{\cos x}{1+\sin x}$ is equivalent to which of the following?

- A) $\cos x + \cot x$ B) $\frac{1}{\sec x + \tan x}$ C) $\frac{1-\sin x}{\cos x}$ D) both A & B E) both B & C

27. A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of drawing a face card or a 6?

- A) 0 B) $\frac{5}{52}$ C) $\frac{4}{13}$ D) $\frac{2}{13}$ E) 1

28. What is the value of $(\log_3 4) (\log_4 5) (\log_5 6) \dots (\log_{80} 81)$?

- A) 4 B) $\frac{1}{3}$ C) $\frac{8}{3}$ D) 2 E) 9

29. *Find the indefinite integral $\int \cos^3 5x \, dx$.

- A) $\frac{\sin 5x(3 - \sin^2 5x)}{15} + C$
B) $\frac{\sin 5x(3 + \sin^2 5x)}{3} + C$
C) $\frac{\sin 5x(3 + \sin^4 5x)}{15} + C$
D) $\frac{\sin 5x(3 - \sin^4 5x)}{15} + C$
E) $\frac{\sin 5x(3 + \sin^2 5x)}{15} + C$

30. *How many positive integers n have the property that when 1,000,063 is divided by n , the remainder is 63?

- A) 79 B) 49 C) 39 D) 37 E) 29

31. Identify the type(s) of symmetry for $x^4y^2 + 2x^2y - 1 = 0$

- A) to x -axis B) to y -axis C) to origin D) both A and B E) none of these

32. Find all vertical asymptotes of the graph of $f(x) = \frac{x-7}{x^2-49x}$.

- A) $x=7, x=-7$ B) $x=-7$ C) $x=7$ D) $x=0, x=49$ E) $x=0, x=7, x=49$

33. The makers of Delight Ice Cream put a coupon for a free ice cream bar in every 80th bar they make. They put a coupon for 2 free bars in every 180th bar and a coupon for 3 free bars in every 300th bar. If they put all three coupons in every n th bar, compute n .

- A) 1200 B) 1800 C) 2400 D) 3600 E) none of these