

4th ANNUAL



FRIDAY, December 7

Individual Exam

Please read each problem carefully. After working the problem, select the correct answer from the five choices and mark that answer on your scantron. In the event of a tie (for individual awards), the following starred (*) test items will serve as tie breakers in the order listed: 2, 5, 8, 9, 12, 17, 18, 21, 24, and 30. There is no correction factor for guessing and any question left blank will be scored as incorrect. You will have 50 minutes to complete the exam. The scantron sheet is machine graded so be sure to use the provided #2 pencil and if you need to change an answer, erase your former selection completely. No calculators allowed.

1. Find all real solutions to the following exponential equation:

$$x^5 e^x + 3x^4 e^x = 4x^3 e^x$$

- A) $(-\infty, \infty)$
- B) $\{-1, 0, 4\}$
- C) $\{0\}$
- D) $\{-4, 0, 1\}$
- E) No solution

*2. Find the inverse of the matrix $M = \begin{bmatrix} -5 & 6 \\ -2 & -3 \end{bmatrix}$.

- A) $\begin{bmatrix} \frac{1}{9} & \frac{2}{9} \\ \frac{2}{27} & \frac{5}{27} \end{bmatrix}$
- B) $\begin{bmatrix} \frac{1}{9} & \frac{2}{9} \\ \frac{2}{27} & \frac{5}{27} \end{bmatrix}$
- C) $\begin{bmatrix} \frac{29}{45} & \frac{2}{9} \\ \frac{10}{27} & \frac{5}{27} \end{bmatrix}$
- D) $\begin{bmatrix} \frac{29}{45} & \frac{2}{9} \\ \frac{10}{27} & \frac{5}{27} \end{bmatrix}$
- E) no inverse

3. Find all values of k for which the given linear system is consistent.

$$\begin{cases} kx + y = -2 \\ 4x - y = 2 \end{cases}$$

- A) $k = 2$
- B) $k = 4$
- C) $k = -4$
- D) The system is consistent for all real values of k .
- E) There is no real value of k which would make the system consistent.

4. If $N > 1$, then $\sqrt[3]{N\sqrt[3]{N\sqrt[3]{N}}} =$

- A) $N^{1/27}$
- B) $N^{1/9}$
- C) $N^{7/27}$
- D) $N^{11/27}$
- E) none of these

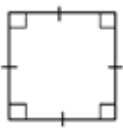

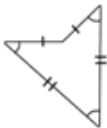

*5. Find the following indefinite integral: $\int \sin x \cos x dx$

- A) $\frac{\sin^2 x}{2} + C$
- B) $\frac{\cos^2 x}{2} + C$
- C) $-\frac{\cos^2 x}{2} + C$
- D) both A) and B)
- E) both A) and C)

6. Add MCDLXIV and MDCCCLVI using only Roman notation. Choose the simplest correct answer.

- A) MMMCCCXX
- B) MMMCCCXVV
- C) MMDDCCCXX
- D) MMMCCLLXX
- E) none of these

7. Which shows a convex, irregular quadrilateral?

- A)  B)  C)  D)  E) none of these

*8. A point is moving along the graph of the function $y = \frac{1}{9x^2+4}$ such that $\frac{dx}{dt} = 2$ centimeters per second.

Find $\frac{dy}{dt}$ when $x = 2$.

- A) $\frac{dy}{dt} = -\frac{9}{5}$ centimeters per second
- B) $\frac{dy}{dt} = \frac{9}{200}$ centimeters per second
- C) $\frac{dy}{dt} = \frac{9}{400}$ centimeters per second
- D) $\frac{dy}{dt} = -\frac{9}{400}$ centimeters per second
- E) $\frac{dy}{dt} = -\frac{9}{200}$ centimeters per second

*9. Find the derivative $f'(x)$ for $f(x) = \cos(\tan^2 x)$.

- A) $-\sin(\tan^2 x)$
- B) $\sin(\tan^2 x)$
- C) $-2 \sin(\tan^3 x) \sec^2 x$
- D) $-2 \sin(\tan^2 x) \tan x \sec^2 x$
- E) None of the above

10. Evaluate the complex number raised to a power and simplify: $(1+i)^{20}$

- A) $-1024i$
- B) -1024
- C) $1024i$
- D) 1024
- E) none of these

11. Find the exact value of the expression $\cos \frac{3\pi}{8}$.

- A) $\frac{\sqrt{2-\sqrt{2}}}{2}$
- B) $\frac{\sqrt{2+\sqrt{2}}}{2}$
- C) $-\frac{\sqrt{2+\sqrt{2}}}{2}$
- D) $-\frac{\sqrt{2-\sqrt{2}}}{2}$
- E) none of these

*12. Find all critical numbers of the function $f(x) = 2 \sec(x) + \tan(x)$, $14\pi < x < 16\pi$.

- A) $x = \frac{91\pi}{6}$
- B) $x = \frac{91\pi}{6}, \frac{95\pi}{6}$
- C) $x = \frac{89\pi}{6}, \frac{95\pi}{6}$
- D) $x = \frac{95\pi}{6}$
- E) $x = \frac{31\pi}{2}$

13. Solve the given equation for x : $\ln x^2 = 2\ln 4 - 4\ln 2$.

- A) $\{ \}$
- B) $\{-1\}$
- C) $\{0\}$
- D) $\{1\}$
- E) $\{-1, 1\}$

14. In the standard (x,y) coordinate plane, where $a \neq 0$ and $b \neq 0$, the graph of $f(x) = \frac{3x+b}{x+a}$ has a horizontal asymptote at:

- A) $y = 3$ B) $y = a$ C) $y = -a$ D) $y = -\frac{b}{2}$ E) $y = \frac{b}{a}$

15. All of the positive integers are written in a triangular pattern, beginning with the following four lines and continuing in the same way:

```

          1
        2 3 4
      5 6 7 8 9
    10 11 12 13 14 15 16
  
```

Which number appears directly below 2012?

- A) 2100 B) 2102 C) 2104 D) 2106 E) 2108

16. What is the value of $\sin \frac{5\pi}{7} + \sin \frac{2\pi}{7} + \sin \frac{12\pi}{7} + \sin \frac{9\pi}{7}$?

- A) 1 B) -1 C) 0 D) $\frac{\sqrt{7}}{7}$ E) $-\frac{\sqrt{7}}{7}$

*17. Find the volume of the solid generated by revolving the plane region bounded by $y = 4x^2$, $y = 10x - x^2$, about the line $x = 2$.

- A) $\frac{40\pi}{3}$ B) $\frac{80\pi}{3}$ C) 40π D) 20π E) $\frac{20\pi}{3}$

*18. Which of the following diverges?

- A) $\sum_{n=1}^{\infty} \frac{3}{4^n}$ B) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ C) $\sum_{n=1}^{\infty} \frac{3}{n^4}$ D) $\sum_{n=1}^{\infty} \frac{3}{4n}$ E) $\sum_{n=1}^{\infty} \frac{3n}{4^n}$

19. Convert the polar equation $r = 10\sin \theta$ to rectangular form.

- A) $6x - y + 5 = 0$ B) $x^2 + y^2 = 25$ C) $x = 5$ D) $x^2 + (y - 5)^2 = 25$ E) $y = 5$

20. Find an equation of the parabola with vertex $(0,7)$ and directrix $y = -10$.

- A) $y^2 = 34(x - 7)$ B) $x^2 = 68(y + 7)$ C) $y^2 = 68(x - 7)$ D) $x^2 = 68(y - 7)$ E) $x^2 = 34(y - 7)$

*21. Use implicit differentiation to find y' if $x^2 + y^2 - 3x + 6y = 9$.

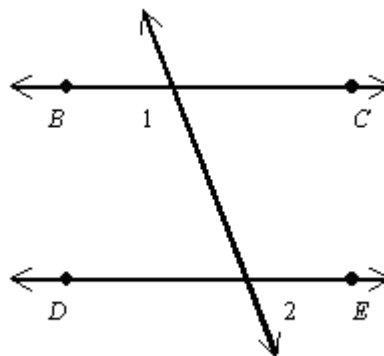
- A) $\frac{2x-3}{2y+6}$
 B) $\frac{3-2x-2y}{6}$
 C) $\frac{12-2x}{2y+6}$
 D) $\frac{3-2x}{2y+6}$
 E) None of the above

22. Which of the following data sets has the greatest standard deviation?

- A) 1,1,1,10,10,10
 B) 3,3,3,3,3,3
 C) 1,2,3,4,5,6
 D) 5,6,7,8,9,10
 E) 2,7,7,10,10,12

23. Find the value of x that will allow you to prove that $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$ if $m\angle 1 = 110^\circ$ and $m\angle 2 = (3x + 4)^\circ$.

- A) 70
 B) $\frac{106}{3}$
 C) 22
 D) 180
 E) none of these



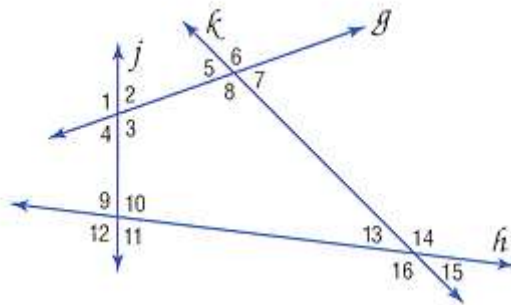
*24. Evaluate: $\int_{-3}^3 (3 - |x|) dx$

- A) 3 B) 9 C) 18 D) 6 E) none of these

25. Find the slant asymptote of $f(x) = \frac{2x^2 - 5x + 7}{x - 2}$.

- A) $y = 2x + 7$ B) $y = 2x - 1$ C) $y = 2x - 5$ D) $y = 2x - 2$ E) $y = 5$

26. Which of the following describes the relationship between $\angle 3$ and $\angle 8$?



- A) corresponding angles
 B) alternate interior angles
 C) alternate exterior angles
 D) consecutive interior angles
 E) choices A) & D)

27. For a positive integer n , define $S(n)$ to be the sum of the positive divisors of n .

Which of the following is the smallest?

- A) $S(2010)$ B) $S(2011)$ C) $S(2012)$ D) $S(2013)$ E) $S(2014)$

28. The distance from Huntsville to Anniston is about 120 miles. If you have two hours to make the trip, but you drive 50 mph for the first 60 miles, how fast must you go on the second half of the trip to arrive in Anniston on time?

- A) 70 mph B) 76 mph C) 80 mph D) 84 mph E) none of these

29. When rewritten as partial fractions, $\frac{3x+2}{x^2-x-12}$ includes which of the following?

I. $\frac{1}{x+3}$ II. $\frac{1}{x-4}$ III. $\frac{2}{x-4}$

- A) I and II B) I only C) II only D) III only E) I and III

*30. Given $x+x^2+x^3+x^4+\dots=1.5$ and assuming that $|x|<1$, solve for x .

A) $\frac{2}{3}$ B) $-\frac{2}{3}$ C) $\frac{3}{5}$ D) $-\frac{3}{5}$ E) none of these

31. What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

- A) 51 B) 50 C) 49 D) 1 E) 0

32. For anterior cruciate ligament (ACL) reconstruction surgery, the probability that the surgery is successful is 0.8. Find the probability that three out of three surgeries are successful.

- A) 0.512 B) 0.336 C) 0.008 D) 0.8 E) none of these

33. You own 13 pair of socks where every pair is different. All of the socks are thrown into a drawer without being tied together. You pull out socks one by one until you have a matching pair. How many socks must you pull out in order to guarantee that you have a matching pair?

- A) 3 B) 12 C) 13 D) 14 E) 25